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Modeling and Valuing Real Options Using Influence Diagrams

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† Comments and suggestions for improvement are welcome and will be gratefully appreciated.

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ABSTRACT

We propose the use of influence diagrams for modeling and valuing real options. Real options are a type of option where the underlying asset is real (as opposed to financial), and they exist when managers have the opportunity, but not the requirement, to alter the firm's operating and/or investment strategy. We use an example from the real options literature to compare and contrast decision trees, binomial trees, and influence diagrams. Specifically, we argue that influence diagrams represent decision problems in a more descriptive, intuitive, and compact manner than decision tree or option-based models. Also, under certain conditions, influence diagram models and option-based models give the same valuations and optimal strategies. Finally, given that the influence diagram decision-making framework can readily be used by corporate managers to model and value real options investment opportunities, this research may provide a means for effectively bringing real options analyses into the applied arena.

Key Words: Capital budgeting, decision analysis, real options, decision trees, binomial trees, influence diagrams

1 INTRODUCTION

Today, most academic researchers, financial practitioners, corporate managers, and strategists realize that when market conditions are highly uncertain, when expenditures are at least partially "irreversible," and when managerial decision flexibility is present, a traditional discounted cash flow (DCF) analysis alone fails to provide an adequate decision-making framework for capital budgeting decisions. This is because a traditional DCF analysis cannot fully account for active project management. It does not properly value management's ability to wait or to revise the initial strategy when future events turn out to be different than expected. Also, management is not obligated to revising the firm's strategy or to undertaking any future discretionary opportunities. Thus the right to do so is an option – a *real option*. In practice, managers decide on capital investments by evaluating, on an on-going basis, their available options to invest in real assets and either exercising them, deferring them, or allowing them to expire.

A great deal of theoretical work has been done on how to model and value managerial flexibility, or *real options*, and derive optimal investment strategies. To date, the primary approach proposed is to use option-pricing techniques, which were originally developed for use with financial assets. Unfortunately, however, these real options models are not widely used by corporate managers and practitioners when making capital budgeting decisions. Lander and Pinches [1998] suggest three reasons why these models are not being used in practice. The first reason is that most corporate managers and practitioners are not familiar with or have a good understanding of the types of models currently proposed. Using continuous-time option pricing techniques to model and value real options requires a relatively sophisticated knowledge of a variety of advanced mathematical techniques, which most corporate managers and practitioners, and even many academics, do not have. Also, although a simple model like a standard Black-Scholes model may not be difficult to actually use, the person doing the analysis still needs to fully understand the assumptions behind the model. There are few easy and direct practical applications of continuous-time option pricing techniques. Binomial models and other discrete-time numerical approximation models are more intuitive and easier to use, but most corporate managers and practitioners have had little experience with these models or how to apply them. Also, many practitioners do not intuitively understand valuing *as if* investors are risk-neutral. The second reason Lander and Pinches [1998] suggest for why the option-based models are not currently being used is that many of the required modeling assumptions are often and consistently violated in real world applications. The third reason is that the necessary additional assumptions required for mathematical tractability limit the usefulness of the models. Many of the real options analyses presented to date, therefore, are relatively straightforward and have been simplified for mathematical tractability. Yet, from a practical standpoint, modeling and valuing real options is not always so easy to do. Real world investment opportunities are often rather complex.

Lander and Pinches [1998] argue that it is not sufficient to show corporate managers and practitioners how to use the currently proposed real options models. They suggest there is a need for more practically oriented modeling frameworks that (1) are easier to understand and use, (2) are not constrained by having to make as many modeling assumptions, and yet (3) can model and value somewhat sophisticated real world investment opportunities. Such models might be more likely to be used. Also, from the applied perspective, the focus of any analysis should be on the decision(s) to be made (i.e., the optimal strategy to follow) and not necessarily on the resulting valuations. In addition, it is important to remember that, in practical real options analyses, option-based models themselves are not completely precise in their valuations because most real assets are not traded assets. Thus, alternative modeling and valuing approaches can be viewed as being consistent with one another as long as the results obtained are similar in terms of the optimal strategy to follow and the general magnitude of value.

This paper introduces influence diagrams as an alternative capital budgeting decision-making framework and compares and contrasts decision tree models, binomial models, and influence diagram models. We argue that influence diagram models are superior to decision tree models, are a relevant and computationally simple approach, and may provide the most practical means for the implementation of modeling and valuing investment opportunities when real options are present. In particular, we argue that (1) influence diagram models represent decision problems in a more descriptive, intuitive, and compact manner than do either decision tree or binomial models, (2) influence diagram models and decision tree models are mathematically equivalent, and (3) for modeling and valuing investment opportunities having real options and under certain conditions, influence diagram models and binomial models give the same valuations and optimal strategies. Corporate managers can easily and readily use the influence diagram decision-making framework to model and value real options investment opportunities. They can also include both the modeling of the estimation process and the modeling of the valuation process. Future research may show influence diagrams can provide a more efficient and tractable method for bringing real options analyses into the applied arena.

The remainder of this paper is organized as follows. In Section 2, we describe a real options decision problem. In Section 3, we describe a decision tree, a binomial tree, and an influence diagram representation and valuation of the decision problem. We also discuss some strengths and weaknesses of each technique. In Section 4, we conclude with a summary of our findings.

2 A REAL OPTIONS DECISION PROBLEM

In order to compare decision tree, binomial option-based, and influence diagram models, a real options decision problem is presented. This decision problem is adapted from an example in Sercu and Uppal's [1995] international textbook *International Financial Markets and the Firm*. Their example is a two-period binomial problem where only one additional unit of a good may be produced, the uncertainty is the future exchange rate, and the firm can initially decide to produce the last additional unit of output, liquidate the manufacturing plant, or defer the production decision until time t = 1. If the firm initially decides to wait, it may at time t = 1, go ahead and produce the one last unit of output or liquidate the manufacturing plant.

In our adaptation, the time frame is three years and one unit of the good may be produced in each period. Although this decision problem is simple, it is sufficient for comparing the basic features of the various decision-making frameworks. And, since there are many downstream decision alternatives at time t = 0, this investment opportunity cannot be adequately modeled using a traditional DCF methods.

2.1 An Investment Problem

An U.S. firm has a three-year investment opportunity where the firm may produce, liquidate, or postpone at both time t = 0 and time t = 1 and produce or liquidate at time t = 2. Postponing the decision at time t = 2 does not make sense as there are no more periods available for production. If the firm wanted to postpone at time t = 2, the preferred decision would be to liquidate for a net positive salvage value, versus postponing for exit costs or not producing for no value. This three-year investment opportunity has the following features:

1. the firm's manufacturing plant is already operating and can produce one unit of output for each of the next three years;
2. at any time, production takes one period and costs $1.80 (USD);
3. at any time, revenue is £1.00 (GBP);
4. it is possible to temporarily exit the market at a cost of $0.10 (USD). Re-entry also costs $0.10 (USD);
5. the net liquidation value of the manufacturing plant at time t = 0, t = 1, or t = 2 is $0.20 ($0.30 salvage value less $0.10 exit costs) (USD) and the salvage value of the plant goes to zero at t = 3;
6. the spot exchange rate is 2 $/£ (USD/GBP) and follows a multiplicative binomial where u = 1.2 and d = 0.8;
7. both the U.S. risk-free rate (r_f(US)) and the U.K. risk-free rate (r_f(UK)) are 5%; and
8. the risky probability of an "up" move in the exchange rate is 0.631 for each period and the risky discount rate for the investment opportunity is 10.5%.

This investment opportunity has multiple real options, besides producing each period. Initially, the firm can decide to produce, liquidate, or postpone the decision until time t = 1. If the firm chooses to liquidate, there is a net positive liquidation value. If the firm chooses to postpone the production decision, it must pay the exit costs. The next year, the firm will then have the right to produce if the exchange rate moves in its favor (i.e., the firm can take advantage of upside potential) but liquidate for a positive salvage value if the exchange rate moves unfavorably (i.e., the firm can alleviate downside loss).

The firm must also pay the exit costs if it initially decides to produce but then subsequently decides to wait at time t = 1. If the firm initially decides to postpone and subsequently decides to wait again at t = 1, it will not owe any additional exit costs. Also, if the firm initially postpones and then produces at t = 1 or if the firm postpones at t = 1 and then produces at t = 2, it must pay re-entry costs along with the period's production cost.

3 DECISION TREES, BINOMIAL TREES, AND INFLUENCE DIAGRAMS

Since influence diagrams can be regarded as compact representations of decision trees, we first describe decision trees. Next we describe binomial trees, which are a variant of decision trees. Finally we describe influence diagrams. We use the real options problem described in the previous section to illustrate all concepts.

3.1 Decision Trees

In general, a decision tree representation of a decision problem has four elements:

1. the possible courses of action or sets of alternatives;
2. the uncertain events with a complete probability model;
3. the resulting outcomes and the net benefit (cost) or preferences of each possible outcome; and
4. the information constraints specifying what uncertain events the decision-maker knows and does not know at each point when a decision has to be made.

In a given decision tree, all possible scenarios must be included. The decision-maker can choose only one alternative (branch) at a decision node. Branches from a chance node must be mutually exclusive and collectively exhaustive. The sequence of decisions in the tree flows from the root to the leaves [Raiffa 1968, Clemen 1991].

The decision tree structure for our example is shown in two parts since it is too extensive to fit on one page. Figure 1a shows the top-half of the decision tree structure. This sub-tree includes the "produce" leg and the "liquidate" leg, but the "postpone" leg has been collapsed. Figure 1b shows the bottom-half. This second sub-tree includes the “liquidate” leg and the “postpone” leg, but the “produce” leg has been collapsed.

Notice that the decision tree representation (Figures 1a and 1b) has 100 nodes, and 53 of them are terminal nodes. Yet, the decision problem has only one source of uncertainty—the exchange rate. This example shows that decision trees can grow exponentially (in the number of decision and chance variables) and may be difficult to present and not easily understood representations of the decision problem.

In this decision tree, “*p*” is the risky or subjective probability of an “up” move in the exchange rate for each period (home currency depreciating) and is equal to 0.631. “1-*p*” is the risky or subjective probability of a “down” move in the exchange rate for each period (home currency appreciating) and is equal to 0.369. Both of these values are given in the problem description. The spot exchange rate is 2 $/£. In one year, according to the problem description, the exchange rate will be 2.4 (i.e., 2 x 1.2) or 1.6 (i.e., 2 x 0.8). Similarly, in two years, the exchange rate will be 2.88, 1.92, or 1.28. Finally, in year three, the exchange rate will be 3.456, 2.304, 1.536, or 1.024. The risky discount rate used for valuation is 10.5%. As an aside, the implied volatility of this investment opportunity is relatively low (18.23%), so a 10.5% risky discount rate for the investment opportunity does not seem unreasonable.

Figures 2a and 2b show the solved decision tree model. The expected value of the produce alternative is $0.795, the liquidate alternative is $0.20, and the postpone alternative is $0.533. Since the maximum NPV is $0.795, the initial optimal action is to produce. The optimal follow-on strategy is to produce as long as the exchange rate depreciates but to liquidate the manufacturing plant the first time the exchange rate appreciates.

3.1.1 Strengths and Weaknesses of Decision Trees

To account for managerial flexibility and future discretionary opportunities, decision trees have been used to “enhance” the traditional DCF analysis [Magee 1964, McCabe and Sanderson 1984]. “Explicit use of the decision-tree concept will help force a consideration of alternatives, define problems for investigation, and clarify for the executive the nature of the risks he faces and the estimates he must make” [Magee 1964, p. 96]. Decision trees can be used to model and value many types of investment projects and are especially useful when uncertainty is resolved at discrete points in time. Decision trees take into account the hierarchical and the sequential nature of investment decisions and force “… management to bring to the surface its implied operating strategy and to recognize explicitly the interdependencies between the initial decision and subsequent decisions" [Trigeorgis 1996, p. 57]. In addition, most corporate managers and practitioners are familiar with decision trees and decision trees are reasonably easy to understand and use. Decision trees have many other strengths as well. They are expressive and flexible. Since they are not constrained by distribution assumptions, they can be used to model any given value or cash flow pattern. A decision tree also maps out all possible alternatives and gives the decision-maker both a project value and an optimal strategy to follow. In addition, a decision tree model can account for variable interdependencies, state and/or time dependent parameters, and path-dependent conditional cash flows such as asymmetric switching costs.

Decision trees are especially useful when modeling foregone earnings and intermediate cash flows is difficult to do in option-based models. Also, a decision tree model may be useful for identifying project specific characteristics and any real options present in an investment opportunity [Kemna 1993], even when an option-based model is used for valuing the investment opportunity. Finally, when an analysis is based on cash flow estimates, a decision tree model and an option-based model have the same data demands. As we will demonstrate shortly, all of the above mentioned strengths are inherited by influence diagrams.

Decision trees, however, have two major weaknesses. First, traditional decision trees use subjective probabilities and a risky discount rate, which captures both time and risk preferences. However, this discount rate may not be appropriate when an option is present. Asymmetric claims on an asset do not necessarily have the same expected rate of return as the asset itself and the appropriate discount rate for the option changes with each change in price of the asset. Thus, when an option is present, the appropriate discount rate for a tree usually is not the same as the discount rate for the underlying asset. As we will see, influence diagrams inherit this weakness.

Yet, decision tree models and binomial option-based models can graphically represent the decision problem the same and decision trees are actually operationally identical to discrete-time option-based models. With option-based models, however, the probabilities have been transformed to allow the risk-free rate to be used as the discount rate. Thus, in theory, decision tree models and binomial option-based models are comparable decision-making frameworks and will give the same set of results "if completely equivalent assumptions are used in each type of analysis" [Teisberg 1995, p. 32]. Unfortunately, however, in order to determine the appropriate or "corresponding" risky discount rate that would make a decision tree analysis equivalent to that of an option-based analysis, it is necessary to already know the true value of the investment opportunity, including the real option(s).

Sensitivity analysis on the discount rate may be a practical way to determine if a given decision tree model is useful or not. If the capital budgeting analysis is not sensitive to the discount rate in the respect that the optimal strategy, or at least the initial decision, would not change for a moderately wide range of discount rates, then the tree may be useful for valuing the investment opportunity. Also, McDonald [1999] concludes “when the timing option is most valuable, it is also least sensitive to deviations from the optimal investment rule. Thus, managers may use approximately correct investment timing rules without losing much value.”

In addition, if market prices are not available, which is common with real assets, option-based models require knowing the appropriate risky discount rate for the underlying asset. If the discount rate is not known, an equilibrium asset pricing model must be used to determine the expected rate of return for the underlying asset. Yet, the appropriate discount rate for the underlying asset, or which asset pricing model to use for determining the rate, is often not obvious [Fama and French 1997, McDonald 1999]. Decision trees, therefore, are not the only real options models subject to valuation error due to the discount rate. Option-based models also require some way of determining any below equilibrium rate of return shortfall (i.e., convenience yield) for the underlying asset.

When dealing with practical real options analyses, it is important to not lose sight of the fact that none of the capital budgeting decision-making frameworks give absolutely precise valuations. Because of this, the decision-maker primarily needs to determine the optimal strategy to follow, or at least the initial decision to be made. Now, it is not surprising when a traditional DCF analysis and a real options analysis gives different optimal strategies. But when do two different real options analyses (e.g., a decision tree model and a binomial model) lead to different optimal strategies? In our experience, the optimal strategy is the same the vast majority of the time if a decision tree approach is employed instead of an option-based approach, even though the specific resulting valuations nearly always are not. This also holds true for the example presented in this paper.

The second major weakness of decision trees is their exponential growth. Decision trees can become burdensome to use as they are inherently combinatorially explosive and can become intractably large and complex. Also, if the joint probability distribution is not described exactly as shown in the tree, we may have to "pre-process" the probabilities before completing the decision tree representation. The preprocessing of probabilities in a multifaceted problem may become intractable when there are many chance variables. Influence diagrams do not suffer from these weaknesses. They are compact and typically do not require pre-processing.

A third weakness of decision trees is the difficulty in determining the subjective probabilities to use with the future uncertain cash flows in each time period. However, if market prices for the underlying asset are not available, this weakness is shared by traditional DCF analyses and may be shared by option-based models as well.

3.2 Binomial Trees

Options are contingent claims and are valued relative to the price of the underlying asset. The option pricing techniques often employed for modeling and valuing investment opportunities having real options are the discrete-time binomial or the continuous-time option pricing techniques, or modifications of either [Black and Scholes 1973, Merton 1973, Margrabe 1978, Cox, Ross and Rubinstein 1979, Brennan and Schwartz 1985, McDonald and Siegel 1986, Pindyck 1991, Dixit and Pindyck 1994, Trigeorgis 1991, 1993, 1996].

Option pricing techniques do not predict future values of the underlying asset. Instead, the future values are assumed to follow some well-defined process. For continuous-time option pricing models, the value of the underlying asset is usually assumed to be lognormally distributed, with normally distributed returns. A geometric Brownian motion is used to model changes in the value of the underlying asset:

$$dS = \mu S\, dt + \sigma S\, dz \qquad (1)$$

where μ is the known and constant expected rate of return, σ is the known and constant volatility, and a standard wiener process (dz) represents uncertainty.

The binomial option pricing model assumes that the value of the underlying asset follows a multiplicative binomial and the up and down parameters (*u* and *d* respectively) and the volatility of the underlying asset (σ) are constant and known. It also uses risk-neutral (or pseudo) probabilities, not subjective probabilities. In the limit, the binomial option pricing model approaches the Black-Scholes model. Both the binomial and continuous-time option pricing models use risk-free rates (not risky discount rates), and assume these rates are constant and known.

Option-based models may be used to value investment opportunities when future market conditions are uncertain but are taken into account in the analysis. That is, they may be used to model and value investment opportunities having real options and to derive at least initial optimal investment strategies. Since managers are not committed to undertaking these real options, the option-based approach to modeling real options is conceptually apropos and a reasonable representation of a managerial decision.

Figures 1a and 1b also show the binomial tree structure for this investment opportunity. Since there is only one 2-state uncertainty in this decision problem and the decision alternatives are explicitly modeled in the tree, there is no difference between the structure of the decision tree and the binomial tree model. The difference between the two models is in the parameter values. Here, "*p*" is the risk-neutral, not risky, probability of an "up" move in the exchange rate (home currency depreciating) and "1-*p*" is the risk-neutral probability of a "down" move in the exchange rate (home currency appreciating). In order to value this binomial model, we must know the values of the *u*, *d*, and *p* parameters. *u* is given to be 1.2, but can be determined by dividing the "up" exchange rate by the spot exchange rate (2.4/2). Although technically *d* is defined as 1/*u*, in this case, *d* is given to be 0.8, and can be determined by dividing the "down" exchange rate by the spot exchange rate (1.6/2). The only parameter we have to determine is *p*, and it can be calculated as follows:

$$p = ((1 + r_f(\mathrm{US}) - r_f(\mathrm{UK})) - d)/(u - d) = (1 - 0.8)/(1.2 - 0.8) = 0.5. \tag{2}$$

The appropriate discount rate to now use in valuing the investment opportunity is the risk-free rate (i.e., r_f(US), which equals 5%).

Figures 3a and 3b show the solved binomial tree. The value of the produce alternative is now \$ 0.69, the liquidate alternative is still \$ 0.20, and the postpone alternative is now \$ 0.438. Since the largest NPV is \$ 0.69, the initial optimal strategy is to produce. The optimal follow-on strategy, again, is to produce as long as the exchange rate depreciates but to liquidate the manufacturing plant the first time the exchange rate appreciates.

The values obtained from a binomial model will usually be different from those obtained from a decision tree model when real options are present. In this case, the resulting values are as follows:

Initial Decision	Decision Tree ($)		Binomial Tree ($)
Produce	0.795	>	0.69
Liquidate	0.20	=	0.20
Postpone	0.533	>	0.438

The value of the liquidate option is the same for both models as no discounting is required for valuing this portion of the tree. The resulting valuations obtained from the binomial model for both the produce and postpone alternatives are less than the values obtained from the decision tree model. This indicates 10.5% is not the true discount rate for this particular tree and, in fact, is too low. Solving for the appropriate discount rates for the sub-trees shows the appropriate discount rate for the "produce" leg is 12.8%, and the appropriate rate for the "postpone" leg is 14.0%. However, the initial and follow-on optimal strategies are the same for both models, and that is what is critical to the decision-maker.

A sensitivity analysis using the discount rate shows postponing is never optimal, producing is optimal as long as the discount rate is less than 26.4%, and liquidating is optimal if the discount rate is greater than 26.4%. Thus the optimal strategy only changes when the discount rate used is more than double either the true rate of 12.8% or the given rate of 10.5%. From a practical standpoint, it does not make a difference whether a decision tree model or a binomial model is used to model and value this decision problem.

3.2.1 Strengths and Weaknesses of Binomial Trees

An option-based approach is specifically designed to model flexibility and has multiple strengths. In general, it can conceptually be used to model and value a variety of types of business decisions, can often be reasonably modified for project specific characteristics, and provides precise and valuable intuition into project valuation. Technically, it is based on theory, introduces asymmetry into the valuation, and models risk directly. Also, under certain conditions, it eliminates having to estimate the expected rate of change in the underlying asset. An option-based model is appropriate when there are sequential projects or multiple phases to a given project. Since volatility is a driver of option value, it is especially appropriate when the volatility of the underlying asset is high. When market prices are available, an option-based model is a robust decision-making framework as (1) traded securities can be used in parameter estimation, (2) subjective probabilities are not needed, (3) future values are not based on projected cash flows, and (4) the appropriate risky discount rate for the underlying asset is most likely not needed.

Yet option-based models have their weaknesses. In general, corporate managers and practitioners are not familiar with these models and often do not have sufficient mathematical training to properly use them. These models quickly become highly complex and computationally demanding and generally cannot account for more than one or two fundamental sources of uncertainty. Technically, option-based models may be sensitive to the process assumed for the future values of the underlying asset. For example, do the future values really follow a lognormal random walk? Even if so, is mean reversion present? Determining the true functional forms of and obtaining realistic estimates for the required modeling parameters can be difficult. This also assumes the pricing formulas can be reasonably modified to account for the parameters. If market prices are not available, the appropriate risky discount rate for the underlying asset or an equilibrium pricing model is required. Also, any shortfall in the required rate of return must be determined. In addition, market prices are required for determining the optimal exercise policy. Finally, option-based models may provide a valuation and an initial optimal strategy but do not necessarily provide guidelines for managing the investment opportunity

Since binomial models can be represented as trees, they are more intuitive and require less mathematical knowledge to build and use. Unfortunately, however, many corporate managers and practitioners have not had much experience using binomial models and struggle with using the risk-free rate as the discount rate. Also, because binomial models are trees, they can quickly grow large, especially if the trees do not recombine.

3.3 Influence Diagrams

Influence diagrams are a type of uncertain reasoning model, as are decision trees, and are a very general modeling technique that can be used to structure, or model, and value investment decision problems. They are a graphical modeling tool (directed acyclic graphs) and provide a framework for systematically analyzing problem situations in which uncertainty plays a major role. Influence diagrams can help reveal important sources of uncertainty and interdependencies that exist in the problem situation and model them in quantitative ways. A given influence diagram represents the underlying structure of a specific decision problem and reflects the decision-maker's knowledge at any given point in time. They are an appropriate decision-making framework when each uncertainty can be modeled by a set of conditional probability distributions (i.e., when each probability model can be broken down into priors and conditionals) [Shenoy 1994]. For complex decision problems, influence diagrams are one method for decomposing the decision problem into smaller, and thus more manageable, components which can be analyzed. They also provide a method for balancing cost-benefit trade-offs and resolving strategy differences when different perspectives, or slight changes in inputs, result in different optimal strategies. Finally, influence diagrams allow for, and may even require, subjective judgments to be included in the analysis. As Howard [1990, p. 3] says, “the influence diagram has proved to be a new ‘tool of thought’ that can facilitate the formulation, assessment, and evaluation of decision problems.”

Influence diagrams were originally developed as “front-ends” for decision trees [Miller, Merkhofer, Howard, Matheson, and Rice 1976, Howard and Matheson 1981]. Since a symmetrical decision tree may be transformed into an influence diagram and an influence diagram and may be transformed into a symmetrical decision tree, solving an influence diagram at that time required that it first be transformed into a symmetrical decision tree, and then the decision tree be solved. A direct method for solving influence diagrams was later developed by Olmsted [1983] and Shachter [1986]. Influence diagrams have also now been adapted for asymmetric decision problems [Smith, Holtzman, and Matheson 1993].

Technically, an influence diagram is a compact representation of a decision tree and consists of two parts. First is the graphical part, or the “picture” (e.g., see Figure 4), and second is the numerical part, or the “tables” (e.g., see Table 1). For an influence diagram to be completely specified, both parts - graphical and numerical - are required.

Like decision trees, influence diagrams have four elements: the decision alternatives, the uncertainties and their distributions, the consequences, and the information constraints. The decisions available to the decision-maker are represented by decision nodes and these nodes are shown as rectangles. In the influence diagram in Figure 4, there are three decision nodes, Decision-year 0, Decision-year 1, and Decision-year 2. At the numerical level, we need to specify the states or alternatives for each decision node.

The uncertainties are represented by chance nodes (ovals) and arcs that point into such nodes. In the influence diagram in Figure 4, there are three chance nodes, Exchange Rate-year 1, Exchange Rate-year 2, and Exchange Rate-year 3. Such nodes and arcs tell us about the qualitative structure of the joint probability distribution for the uncertainties. The node at the *start* of an arc is called a predecessor node and the node at the *end* of an arc is called a successor node. That is, given A → B, A is the predecessor node and B is the successor node. The lack of an arc between two chance nodes signifies conditional independence. For example, in Figure 4, the lack of an arc from Exchange Rate-year 1 to Exchange Rate-year 3 implies that these two chance variables are conditionally independent given Exchange Rate-year 2. At the numerical level, for each chance node we need to specify a set of possible states and a set of conditional probability distributions, one for each state of its predecessors.

Deterministic nodes (double ovals) are a special type of chance node in that they have a degenerate conditional probability distribution. That is, given the states of its predecessors, there is only one possible state for the deterministic node. Consequently, this state has probability one. Since the deterministic node is conditionally deterministic, there is no need to specify the conditional probability distribution for it. However, for each deterministic node, we must specify a mathematical function that defines the state of the deterministic node as a function of the states of its predecessor nodes. Also, if any predecessor nodes are chance nodes, then the state it represents is uncertain. Deterministic nodes, although not required in the modeling process, are useful for depicting the decision problem more descriptively. In the influence diagram in Figure 4, there are four deterministic nodes, Cash Flow-year 0, Cash Flow-year 1, Cash Flow-year 2. and Cash Flow-year 3.

The third element of the graphical part of an influence diagram are the information constraints, which specify what the decision-maker knows and does not know at the time a decision is to be made. Arcs that point into decision nodes represent these information constraints. Such arcs are called *sequence* arcs and indicate that the true states of the predecessor nodes will be known at the time the decision has to be made. The absence of an arc pointing from a chance node into a decision node indicates the true state of the chance node will not be known at the time of the decision. For example, in the influence diagram in Figure 4, when the decision in year 0 has to be made, none of the true states of exchange rates are known. However, for the decision in year 1, the true state of exchange rate in year 1 is known and the true states of exchange rates in year 2 and 3 are unknown, and for the decision in year 2, the true states of exchange rates in year 1 and 2 are known and the true state of exchange rate 3 is unknown.

The fourth, and final, element of an influence diagram is the consequences, which are represented by value nodes. Value nodes are shown as rounded rectangles. In the influence diagram in Figure 4, there is one value node (labeled NPV). For each value node, a payoff function that depends on the states of the predecessor nodes, must be specified. As with deterministic nodes, if an arc points into a value node then the predecessor node is *relevant* for the determination of the final value and the arcs signifies the domain of the payoff function. Although an influence diagram may have more than one value node, for this paper, we limit our influence diagrams to having only one value node.

An influence diagram is solved by sequentially deleting nodes. The information constraints dictate the deletion sequence. A maximization or minimization process, depending on the objective of the decision problem, is used to delete decision nodes. They also are deleted in reverse order. In other words, the last decision node is deleted first, the next to last decision node is deleted second, and so on until all decision nodes have been deleted. Chance nodes are deleted by calculating expected values. Chance nodes whose true states will not be known at the time a decision has to be made must be deleted before the decision node can be deleted. Chance nodes whose true states will be known at the time a decision has to be made are deleted after the decision node is deleted. This deletion process continues until only the value node remains. The value node then provides the decision-maker with the resulting expected payoff and the optimal strategy can be constructed from the details of the deletion of the decision nodes.

Influence diagrams may be an alternative framework for modeling and valuing real options [Lander 1997]. An influence diagram graphical representation of this three-year decision problem is shown in Figure 4, and the numerical part of the influence diagram is detailed in Table 1. Using the same basic information as the decision tree model and the binomial model, the initial and follow-on optimal strategies determined by and the resulting valuations obtained from the influence diagram model are (1) the same as those of the decision tree model when p = 0.631 and the discount rate used in valuing the investment opportunity is 10.5% or (2) the same as those of the binomial model when p = 0.5 and the discount rate used in valuing the investment opportunity is 5%. Thus an influence diagram model can emulate either a decision tree model or a binomial model, depending on the inputs. Yet, if the decision tree inputs are used, we see that the influence diagram valuations are also subject to the discount rate criticism.

The ability of an influence diagram to represent a decision problem in a more conceptual or descriptive and yet compact manner is readily seen. Comparing Figures 1a and 1b to Figure 4 shows it is still easy to "tell a story" from the influence diagram and to see the relationships between the decision problem's variables, but that it is not easy at all to do so with the decision tree model. The influence diagram has only 11 nodes, whereas the decision tree has 100 nodes.

3.3.1 Strengths and Weaknesses of Influence Diagrams

For modeling and valuing investment opportunities having real options, influence diagrams have the same strengths as decision trees. However, in addition, influence diagrams can model continuous variables and can represent "relationships between variables equally well whether those variables are discrete, continuous, or a mixture of both" [Smith 1989, p. 363]. They have a more efficient solution procedure than do decision trees. The solution procedure for influence diagrams takes advantage of conditional independence and then can (1) allow for local computations to compute the required conditionals and (2) use local computations on the space of chance variables and on the joint space, as well as on the space of strategies. No preprocessing of the probabilities is required if there are conditionals for each chance node [Shenoy 1994]. Evidence propagation, where the probability distribution of one or more of the uncertainties is updated to reflect the revised knowledge of the decision-maker, is easy to do in an influence diagram model. Thus, from a modeling perspective, project monitoring is a relatively straightforward process and generally does not require the development of new models.

Most importantly, although influence diagrams and decision trees are mathematically equivalent, influence diagrams do not graphically display all possible scenarios. Influence diagrams represent "relationships between the problems component decision variables and random vectors, rather than the relationships between each possible combination of decision and outcome that might occur" [Smith 1989, p. 363]. Thus, in an influence diagram, independence between events can be more easily seen and the size of an influence diagram grows linearly in the number of variables, versus exponentially as with decision trees. The relevant problem details are present in an influence diagram but they are represented locally.

As shown in our decision problem, influence diagram models can be a comparable decision-making framework to both the decision tree and option-based decision-making frameworks for modeling and valuing investment opportunities having real options. Since influence diagrams and decision trees are mathematically equivalent representations of a decision problem [Howard and Matheson 1981], a correctly specified influence diagram model will give the same results as a decision tree model, and vice versa. Also, under certain conditions, a decision tree model will give the same results as an option-based model [Teisberg 1995, Smith and Nau 1995]. Then it follows that, under the same conditions, an influence diagram will also give the same results as an option-based model.

Influence diagrams, as with decision trees, also may be useful for modeling the investment opportunity when combined with option pricing techniques for valuing the investment opportunity. In other words, management could use the graphical part of an influence diagram to explore the features and characteristics of an investment opportunity and to develop a fuller understanding of the investment opportunity and the decision(s) to be made. Then, option pricing techniques could be used for determining the optimal strategy and for valuing the investment opportunity. Thus, given an influence diagram's ability to better represent the decision problem, an influence diagram model may be extremely useful even when option pricing techniques are used for valuation.

As with any decision-making framework, however, influence diagrams have a set of weaknesses. First, they require each uncertainty be modeled by a set of conditional probability distributions [Shenoy 1994] and dummy states be used when the decision problem is asymmetric [Bielza and Shenoy 1996]. In order to be solved exactly, the chance and decision variables all have to have discrete state spaces. Also, because influence diagrams have a more efficient solution procedure than that for decision trees and since they grow linearly in the number of variables, using influence diagrams does address the exponential growth problem of decision trees. But, influence diagrams still suffer from the issue of the appropriate discount rate when an option is present. As previously mentioned, however, a NPV profile will show the range of discount rates where the optimal strategy (or at least the initial decision) does not change, option value is more robust to errors in the investment rule as option value increases [McDonald 1999], and option-based models are also subject to discount rate errors when market prices are not available [Fama and French 1997, McDonald 1999]. Finally, when the underlying asset is not a traded asset, influence diagram models share the problem of estimating the probabilities of the future uncertain cash flows with traditional DCF analyses, decision tree models, and possibly option-based models.

4 SUMMARY AND CONCLUSIONS

In our decision problem, we see the basic similarities and differences of the various decision-making frameworks.

1. The decision tree and binomial tree are structurally the same when there is one 2-state uncertainty and the decision alternatives are explicitly included in the model. Both of these models grow exponentially as variable and time periods are added to the model. Therefore, they are unsuited for modeling large problems.
2. The solved decision tree and the solved binomial tree yield different valuations due to the different asset pricing models assumed and, accordingly, the different probabilities and discount rates used in valuing the investment opportunity.
3. Even with different resulting values, the optimal strategies (initial and follow-on actions) are not necessarily different for the two models.
4. Depending on the inputs, an influence diagram model can emulate either a decision tree model or a binomial option-based model and can be an alternative decision-making framework for investment opportunities having real options.
5. Influence diagrams are superior in how they graphically represent and present decision problems. They are more descriptive of the decision problem, better illustrate the relationships between and among the decision problem's variables, and grow linearly as time periods and variables are added to the model.

This decision problem illustrates that an influence diagram model is more easily implemented for real options analyses and can readily be used by corporate managers to value investment opportunities having real options. Practically, therefore, this research may provide a means for bringing real options analyses into the applied arena.

Yet, influence diagrams have other advantages not shown by this particular example, and this is an area for future research to demonstrate. First, due to the lack of restrictive modeling assumptions and their ability to grow more slowly, influence diagram models can better handle decision problems where there are multiple uncertainties and many time periods. In the decision problem presented here, the number of units produced, revenue, liquidation values, and costs (production, exit, and entry) are all known and constant. These assumptions may not be reasonable. And the probability for each period of an "up" move (p) in the exchange rate is assumed to be the same and is not dependent on the type of move that occurred last period.

These assumptions may not be reasonable either. It may be more likely for the exchange rate to depreciate (appreciate) in period t + 1 if it depreciated (appreciated) in period t. Also, assuming a binomial distribution for the exchange rate limits the exchange rate to only moving up or down in the next period, and at the same rate of change (i.e., u and d are constant).

Consider the following decision problem. The time horizon is four years and produce, liquidate, or postpone decisions need to be made each period. There are a total of six uncertainties modeled. The first three uncertainties are relevant during periods 1, 2, and 3. Of these three uncertainties, two have trinomial distributions and one has a binomial distribution. The last three uncertainties are relevant during period 4 and, again, two have trinomial distributions and one has a binomial distribution. Also, for each period, the probabilities for at least one uncertainty are conditioned on the previous outcome. A compactly designed tree model for this decision problem has over 42,600 nodes. Yet one possible influence diagram graphical representation of this decision problem has a total of only 22 nodes (of which 8 are deterministic nodes, which can be omitted). Also, option-pricing techniques cannot handle this many uncertainties unless only the resulting net cash flow or value per period is modeled as the uncertainty.

For a second example, again consider the decision problem presented in this paper. It would be more reasonable to assume that exchange rates are a function of some set of macroeconomic variables (instead of assuming that exchange rates follow a random walk over time). For example, the exchange rate could be modeled as a 2-state variable, but as a function of the U.S. interest rate, the U.K interest rate, and the spot exchange rate, and according to an interest rate parity or no-arbitrage relationship. In such a model, the exchange rate does not follow a well-defined binomial process. Decision tree models can handle any uncertainty model, and so can handle this easily. Binomial trees cannot handle this as easily, however, as now the u, d, and p values for each time period and node may well be different, violating the underlying assumptions of a binomial model. An influence diagram model can also easily handle this model for the exchange rate. In addition, since influence diagrams grow more slowly as time periods and variables are added, influence diagrams can easily include the modeling of the estimation process as well as the modeling of the valuation process. Thus, not only can influence diagram models handle more complex decision problems but they can also more thoroughly model decision problems.

Finally, since the solution procedure for influence diagram models can take advantage of conditional independence and can solve decision problems using local computations, they may not require the joint probability distribution. Therefore, in addition to being well suited for modeling and valuing traditional investment opportunities having real options, influence diagrams models may also be well suited both for modeling complex decision problems and for solving such complex decision problems, like those found in more real world or sophisticated capital budgeting decision-making.

In conclusion, influence diagram models allow for the development of richer models of uncertainty. They can easily model multiple uncertainties, may include the modeling of any required estimation processes, can model both continuous distributions and discrete distributions or combinations of each, and take advantage of conditional independence and solve the decision problem using local computations. Since real world investment opportunities are often complex due to the number and modeling of uncertainties, influence diagram models may prove to be a more efficient and tractable method for modeling and valuing such problems, which today are impossible to model and value using the current decision-making frameworks.

However, none of the capital budgeting decision-making frameworks discussed in this paper is without implementation difficulties. Each decision-making framework requires at least one input that is difficult to estimate and each has its own set of weaknesses. Sometimes the choice of which decision-making framework to use may be obvious. Usually, however, there is no one ideal decision-making framework to always use. This is especially true when managerial flexibility is valuable, the optimal strategy is sensitive to the risky discount rate used in valuing the investment opportunity, and there are “modeling problems” with using option pricing techniques. However, we believe this is when influence diagram models may be the most useful, thus bringing real options analyses into widespread practice.

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Figure 1a. Top-Half of Decision/Binomial Tree Structure

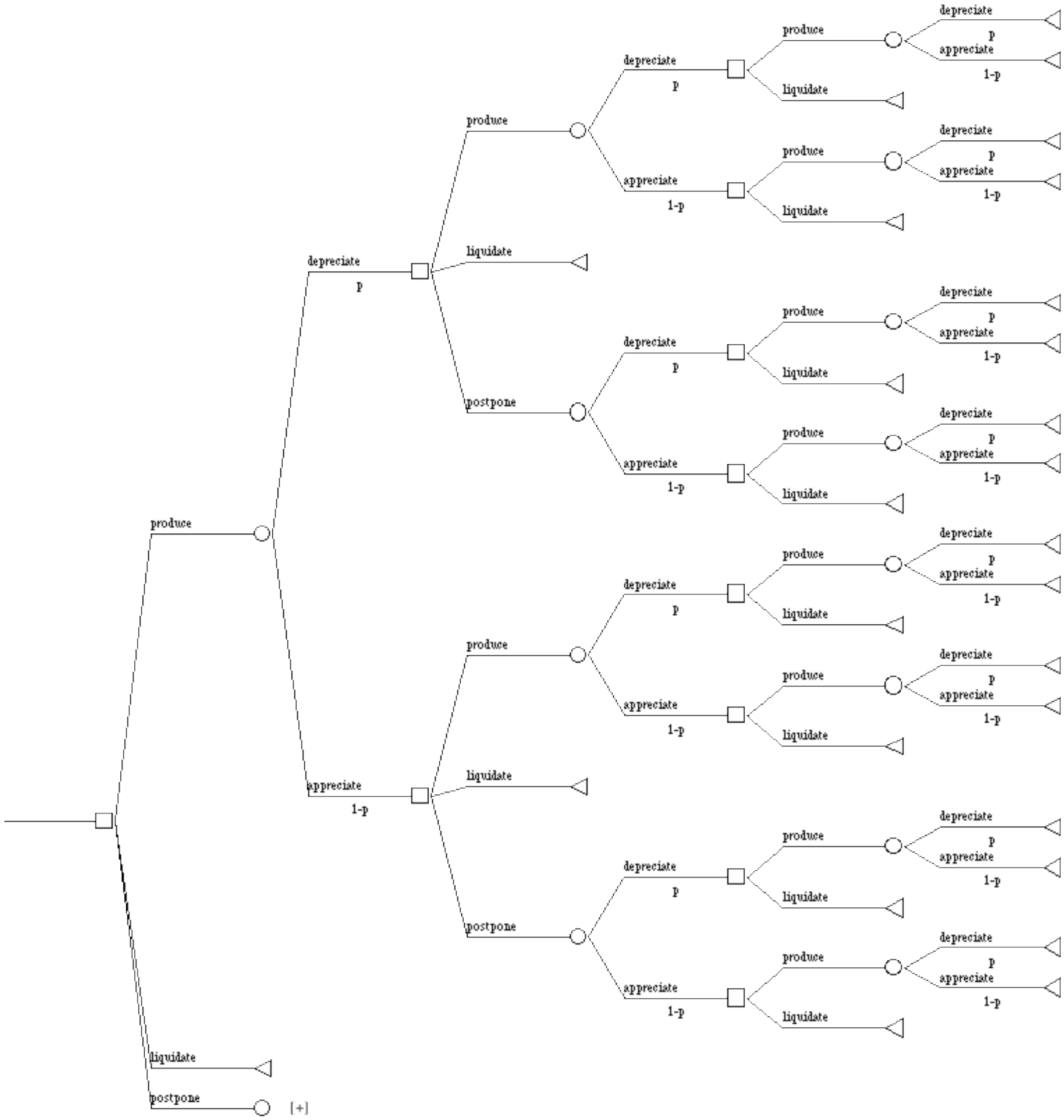

Figure 1b. Bottom-Half of Decision/Binomial Tree Structure

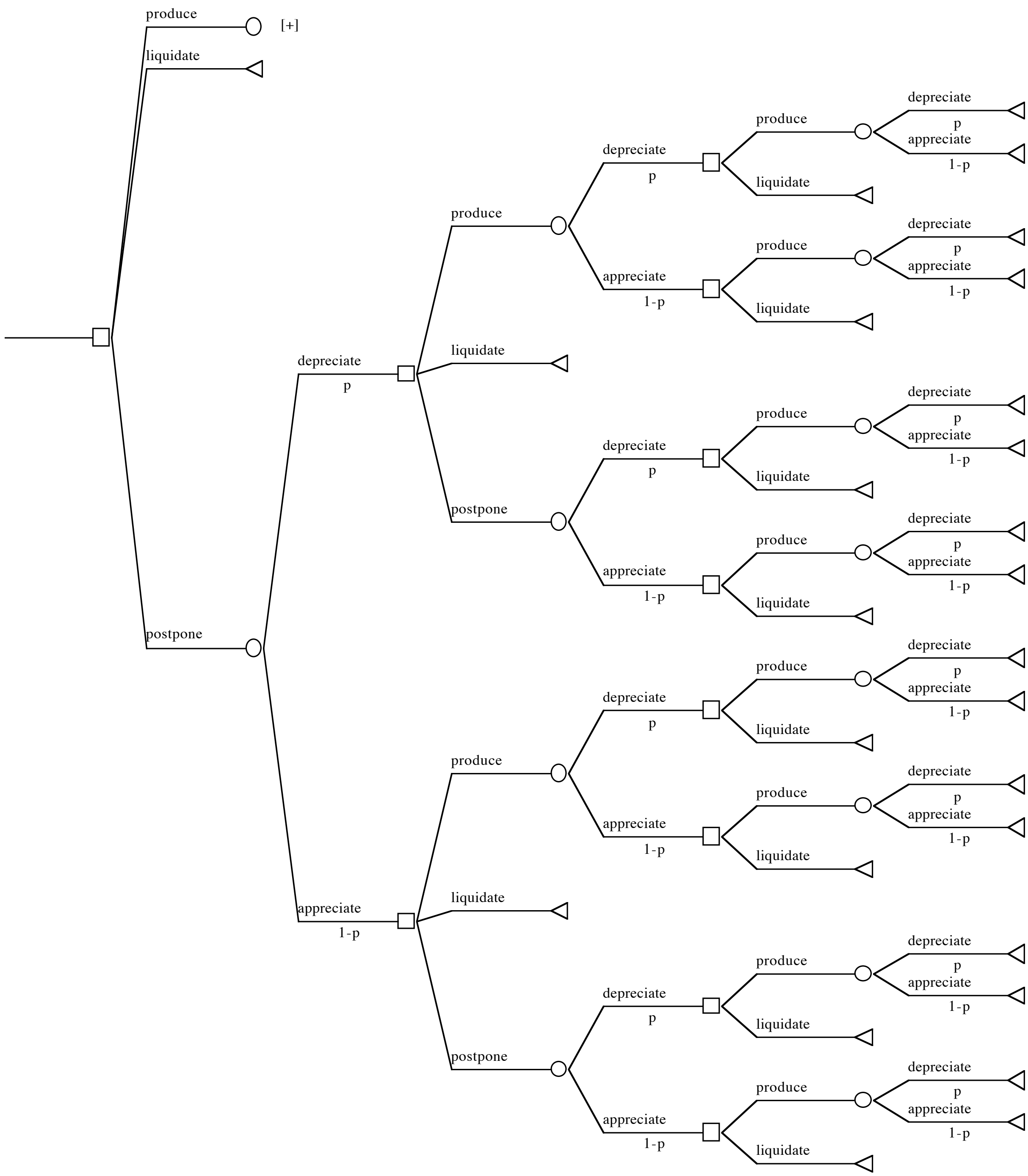

Figure 2a. Top-Half of Solved Decision Tree

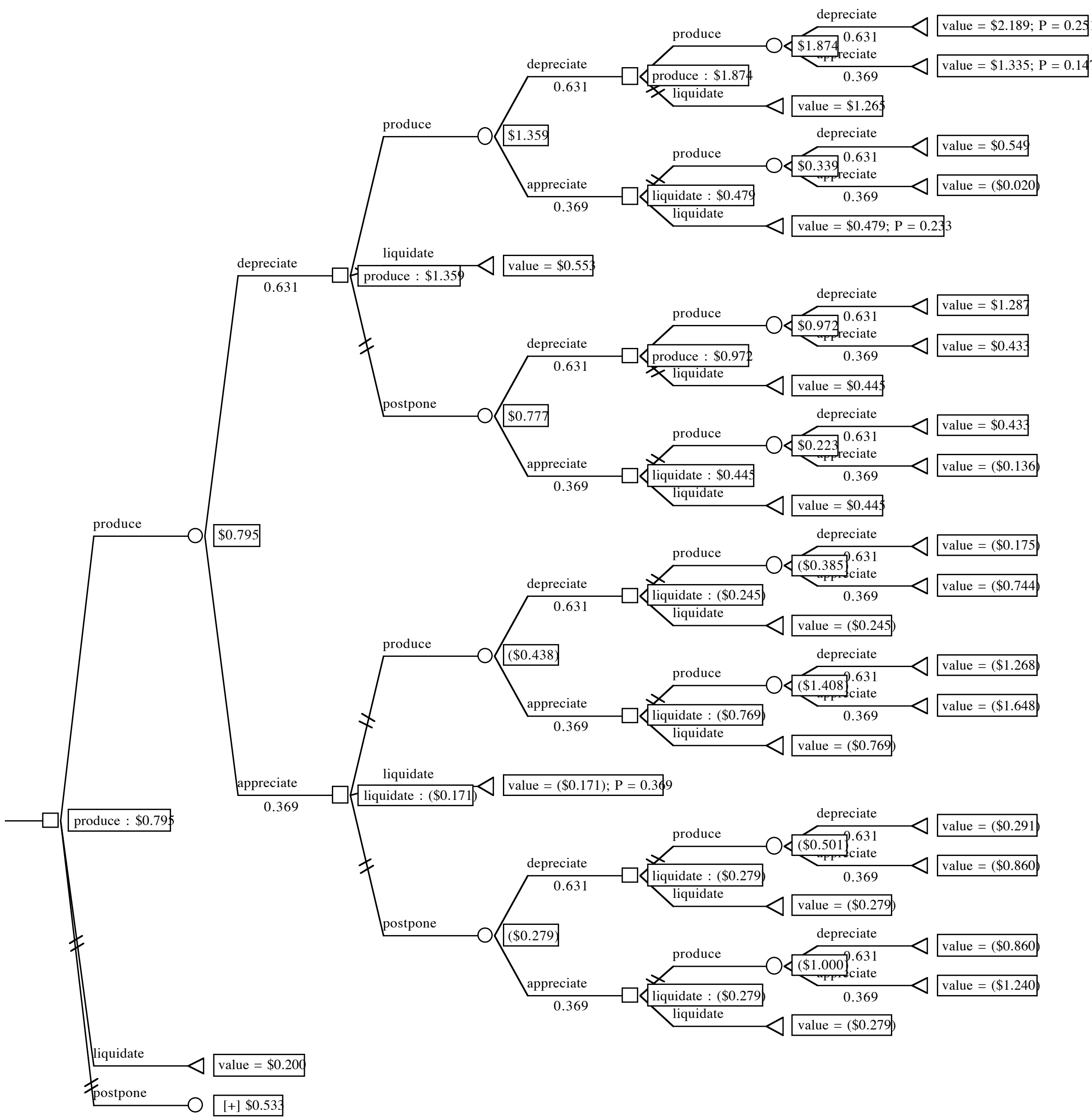

Figure 2b. Bottom-Half of Solved Decision Tree

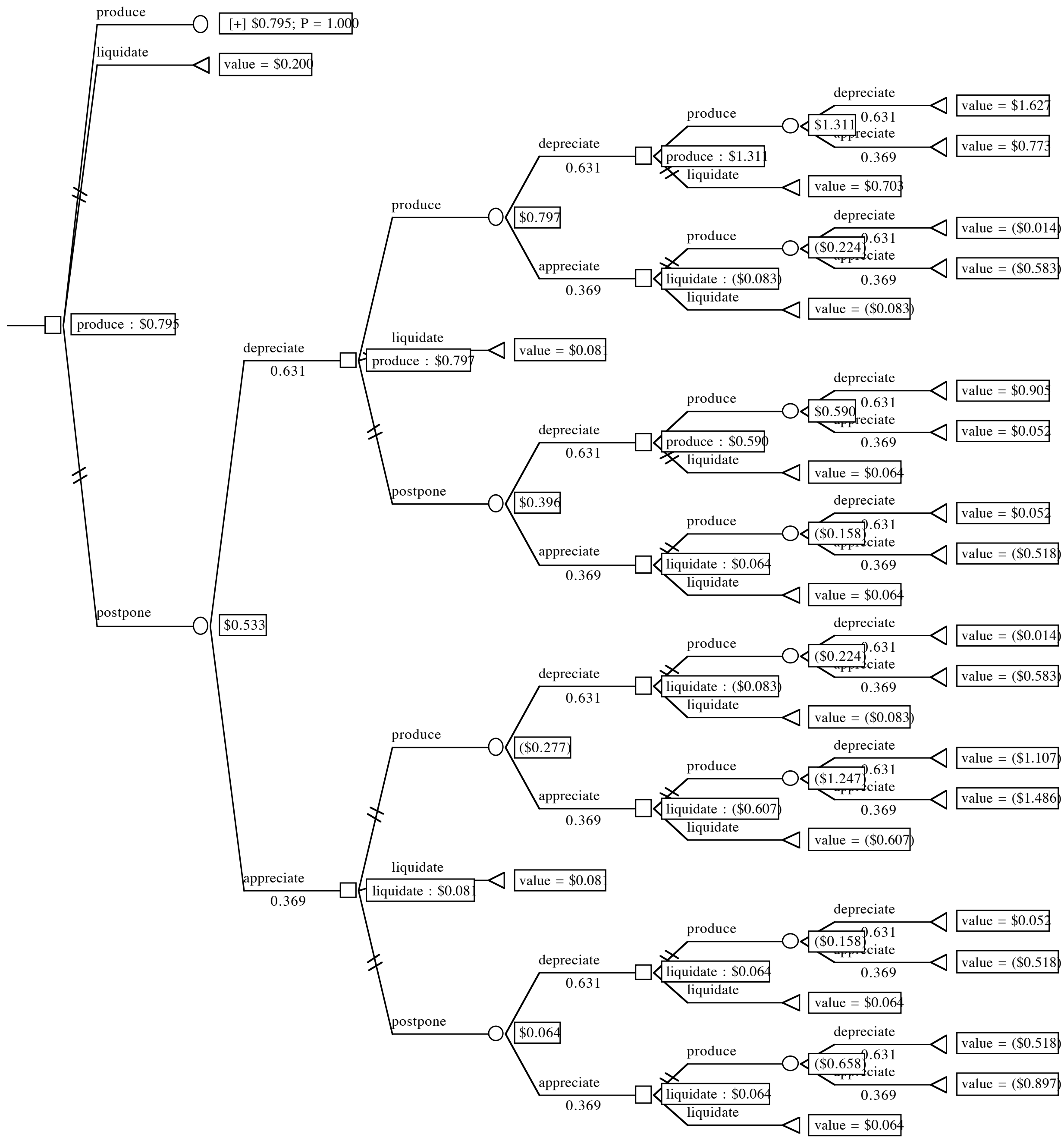

Figure 3a. Top-Half of Solved Binomial Tree

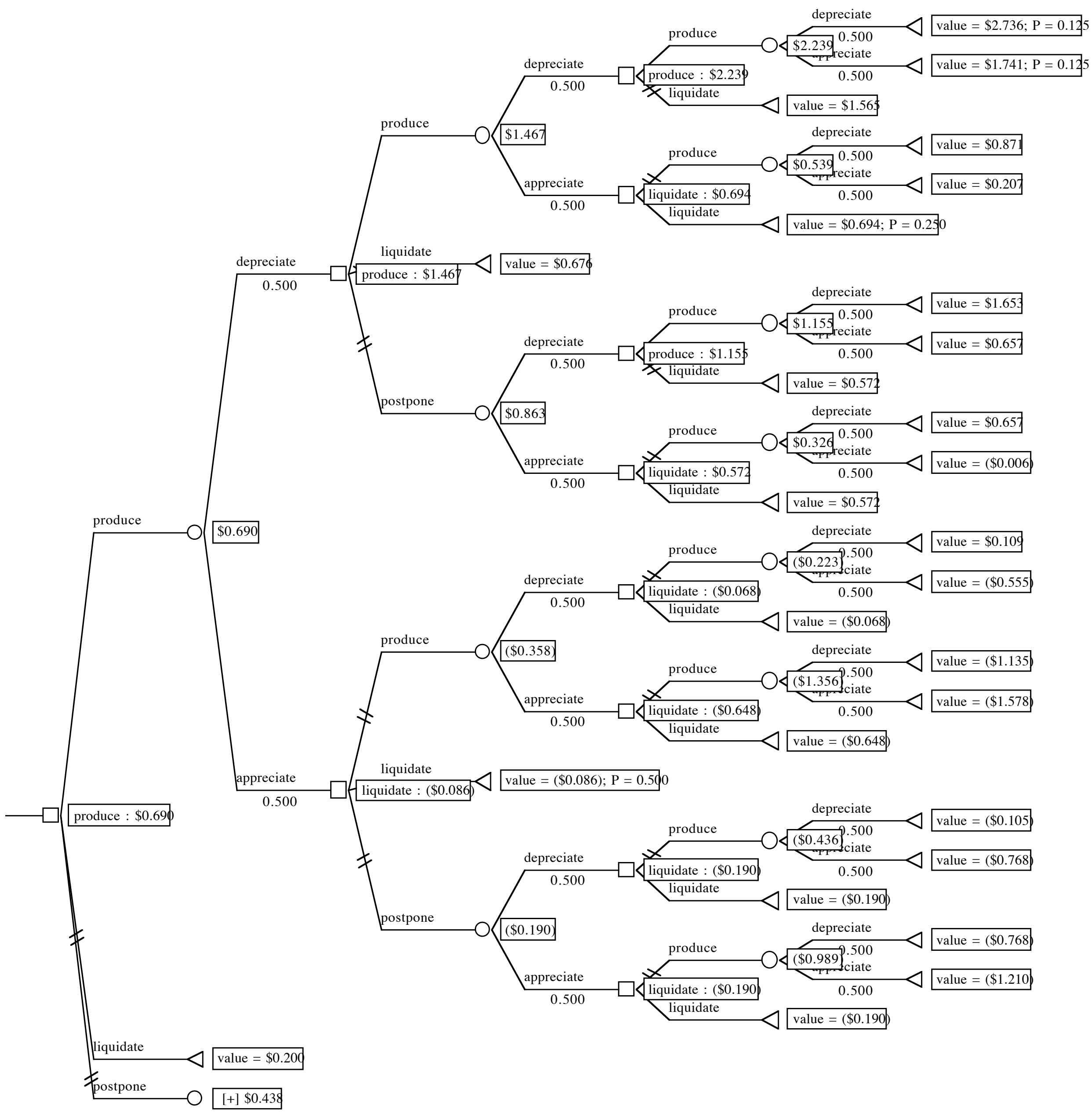

Figure 3b. Bottom-Half of Solved Binomial Tree

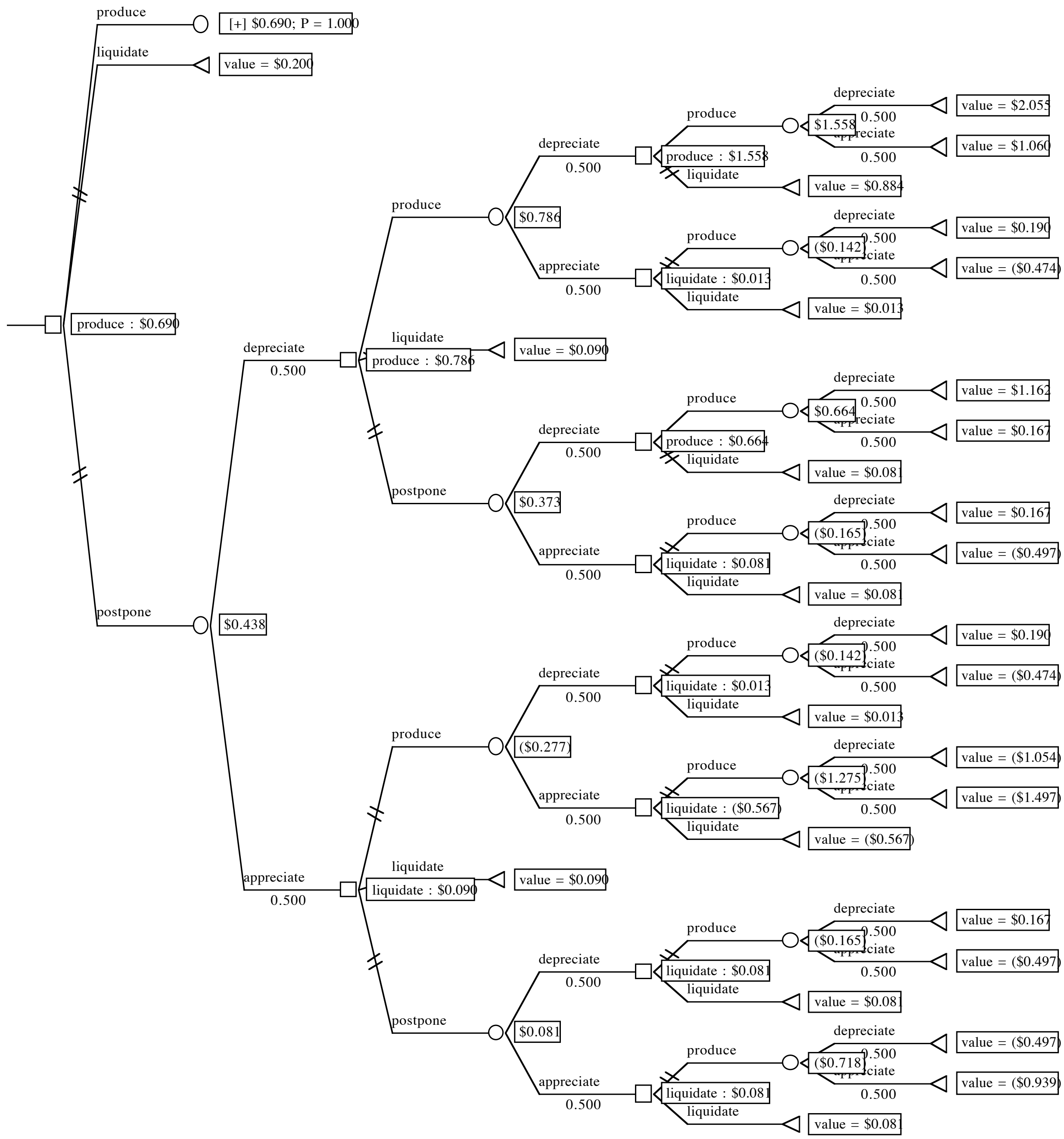

Figure 4. Influence Diagram Graphical Representation

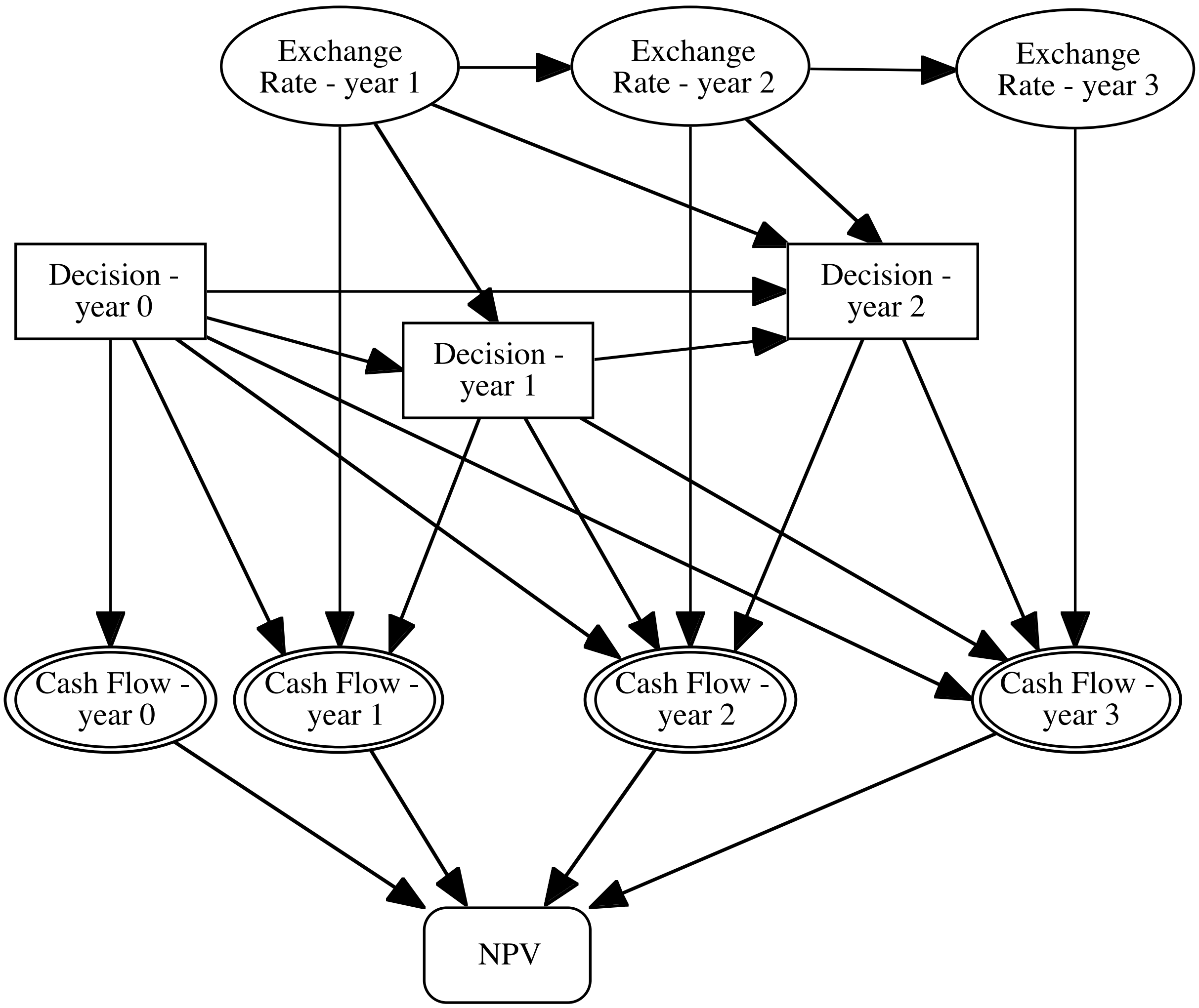

Table 1. Numerical Information for the Influence Diagram in Figure 4

Variable Name (symbol)	*States* (symbols)	*Probabilities/Values*			
Decision - yr. 0 (D_0)	produce (p_0), liquidate (l_0), postpone (d_0)				
Decision - yr. 1 (D_1)	produce (p_1), liquidate (l_1), postpone (d_1)				
Decision - yr. 2 (D_2)	produce (p_2), liquidate (l_2)				
Exchange Rate - yr. 1 (C_1)	($) 2.4 1.6	C_1		P(.)	
		2.4 1.6		p[a] 1–p	
Exchange Rate - yr. 2 (C_2)	($) 2.88 1.92 1.28	C_2		$P(.\|C_1=2.4)$	$P(.\|C_1=1.6)$
		2.88 1.92 1.28		p 1–p 0	0 p 1–p
Exchange Rate - yr. 3 (C_3)	($) 3.456 2.304 1.536 1.024	C_3	$P(.\|C_2=2.88)$	$P(.\|C_2=1.92)$	$P(.\|C_2=1.28)$
		3.456 2.304 1.536 1.024	p 1–p 0 0	0 p 1–p 0	0 0 p 1–p
Cash Flow - yr. 0 (V_0)	($)	D_0		V_0	
		P_0 l_0 d_0		–1.8 0.2 –0.1	
Cash Flow - yr. 1 (V_1)	($)	D_0, D_1		V_1	
		p_0, p_1 p_0, l_1 p_0, d_1 l_0, *[b] d_0, p_1 d_0, l_1 d_0, d_1		$C_1 - 1.8$ $C_1 + 0.2$ $C_1 - 0.1$ 0 –1.9 0.2 0	

(continued on next page)

(continued from previous page)

Variable Name (symbol)	*States* (symbols)	*Probabilities/Values*	
Cash Flow - yr. 2 (V_2)	($)	D_0, D_1, D_2	V_2
		p_0, p_1, p_2	C_2–1.8
		p_0, p_1, l_2	C_2+0.2
		$p_0, l_1, *$	0
		p_0, d_1, p_2	–1.9
		p_0, d_1, l_2	0.2
		$l_0, *, *$	0
		d_0, p_1, p_2	C_2–1.8
		d_0, p_1, l_2	C_2+0.2
		$d_0, l_1, *$	0
		d_0, d_1, p_2	–1.9
		d_0, d_1, l_2	0.2
Cash Flow - yr. 3 (V_3)	($)	D_0, D_1, D_2	V_3
		p_0, p_1, p_2	C_3
		p_0, p_1, l_2	0
		$p_0, l_1, *$	0
		p_0, d_1, p_2	C_3
		p_0, d_1, l_2	0
		$l_0, *, *$	0
		d_0, p_1, p_2	C_3
		d_0, p_1, l_2	0
		$d_0, l_1, *$	0
		d_0, d_1, p_2	C_3
		d_0, d_1, l_2	0
NPV[c] (NPV)	($)	$V_0 + (V_1 / (1 + DR)) + (V_2 / (1 + DR)^2) + (V_3 / (1 + DR)^3)$	

Notes.

a. For solving the influence diagram, p = 0.631 when using the decision tree parameters and p = 0.5 when using the binomial parameters.

b. A $*$ here indicates the decision alternative(s) omitted give the same values.

c. For solving the influence diagram, DR = 10.5% when using the decision tree parameters and DR = 5% when using the binomial parameters.

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