

# Belief Functions and Belief Maintenance in Artificial Intelligence

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## INTRODUCTION

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The idea for this special issue first arose at the Fourth Workshop on Uncertainty in Artificial Intelligence (AI) held at Minneapolis, Minnesota, in July 1988. Jim Bezdek, the editor-in-chief of this journal, asked us if we were willing to act as guest editors. We thought it was a great idea for two reasons: (1) There was (and still is) considerable interest in the AI community on the Dempster-Shafer theory of belief functions, and (2) members of the AI community seemed to like the basic premises of the belief function approach but were not quite sure how to use them in real applications.

As first-time guest editors usually do, we made up an ambitious schedule, which has since slipped quite a bit. However, thanks to Jim Bezdek, we will still make the targeted 1990 publication date.

A lot of our gratitude in making this issue a reality goes to the referees for their thorough reviews and prompt responses. We thank them by name at the end of this introduction. With their help, we accepted the papers by Tom Strat, Didier Dubois and Henri Prade, and Greg Provan—for inclusion here. In addition, we invited Glenn Shafer and Judea Pearl to write position papers that would be published unrefereed. Both kindly consented to do so and, as this issue testifies, delivered on their promises. These two papers make significant contributions to the state of the art in belief function theory, semantics, and application.

The most comprehensive source on the theory of belief functions is still Shafer's 1976 monograph, *A Mathematical Theory of Evidence* [1]. However, that work has little about interpretation, implementation, or computation, topics of great interest to AI researchers. Much has been written on these subjects over the fifteen years since the monograph appeared. In his paper in this issue, "Perspectives on the Theory and Practice of Belief Functions," Shafer attempts to summarize his views on these subjects. As is evident from Pearl's article, which follows Shafer's in these pages, there is still considerable argument about the correct interpretation of belief functions. Shafer's article should go a long way toward clarifying some of these issues. The bibliography of about 140 articles will also be very useful to researchers on this subject.

Pearl, in his paper "Reasoning with Belief Functions: An Analysis of Compatibility," raises fundamental issues regarding the meaning and expressiveness of belief functions in three crucial reasoning tasks: (1) representation of incomplete knowledge; (2) method for belief updating, and (3) evidence pooling. However, in light of other known viewpoints (Smets [2] and Shafer, this issue), we feel that these arguments are still inconclusive. For example, from a pragmatic standpoint, it is not clear whether the debate between "the reliability of evidence in making conclusions" interpretation (Shafer [3]; Shafer and Tversky [4]) and the "the probabilities of provability" interpretation (cf. Pearl and Provan, this issue) is really resolved.

Undoubtedly, Pearl makes a number of good points, and it is very important that practitioners of belief functions demonstrate how the semantics or the ontology of belief functions apply to their particular problem-solving domain. We feel it is very important that the debate on the meaning and applicability of belief functions continue. Therefore, a forthcoming special issue of this journal will contain solicited responses to both Shafer's and Pearl's papers, and both Shafer and Pearl will have an opportunity to reply to them in that issue.

Strat's paper is titled "Decision Analysis Using Belief Functions." As the title indicates, Strat generalizes Bayesian decision theory to situations where uncertainty is represented by a belief function instead of a probability distribution. Strat provides a definition of an expected value of a belief function that is a convex combination of the upper expected value and a lower expected value. We note that Jaffray [5] has defined a similar expected value. The only difference between Strat and Jaffray is that Strat assumes point-valued utilities and models uncertainty as belief functions whereas Jaffray models utilities as belief functions but assumes point-valued probabilities. Mathematically, the two are equivalent.

Dubois and Prade's paper is titled "Consonant Approximations of Belief Functions." Consonant belief functions are of interest to fuzzy-set theorists because a membership function can be regarded as a consonant belief function. These authors describe two methods for approximating a belief function by a fuzzy set; inner approximation that provides an upper bound on beliefs and an

outer approximation that provides a lower bound on beliefs. This paper will help bridge some of the gap between the theories of belief functions and those of fuzzy sets.

Provan's paper, "A Logic-Based Analysis of Dempster-Shafer Theory," develops an elaborate framework for linking DS theory with propositional logic by establishing correspondences between logical relations and set theory operations. This framework provides the basis for analysis of the computational complexity of the evidence combination mechanisms. Like Pearl [6] and Laskey and Lehner [7], Provan establishes a correspondence between the ATMS formalism and DS theory. A major contribution of this work is the link it establishes between logic and belief functions and, like Pearl ([6] and this issue), casts belief functions in the "probability of provability" framework. How useful this formulation will be in advancing belief function applications remains to be seen.

In our request for papers we said that our goal was to produce an outstanding issue that would be treasured by researchers in AI and allied disciplines. We hope we have accomplished our objectives. We would like to thank all authors who submitted papers for consideration for the special issue. Even though we were unable to accept them all, we hope the referees provided constructive feedback. Several of these papers merited publication and are likely to appear in regular issues of this journal.

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